

# Statistical Inference for Multilayer Networks in Political Science\*

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## Abstract

Interactions between units in political systems often occur across multiple relational contexts. These relational systems feature interdependencies that result in inferential shortcomings and poorly-fitting models when ignored. General advancements in inferential network analysis have improved our ability to understand relational systems featuring interdependence, but developments specific to working with interdependence that cross relational contexts remain sparse. In this paper, I introduce a multilayer network approach to modeling systems comprising multiple relations using the exponential random graph model (ERGM). In two substantive applications, the first a policy communication network and the second a global conflict network, I demonstrate that the multilayer approach affords inferential leverage and produces models that better fit observed data.

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# 1 Introduction

Many of the political phenomena we are interested in are outcomes of interactions between units in political systems. These relational systems are complex, featuring interdependence among units that cannot be appreciated when the system is reduced to independent components. Network approaches to modeling provide useful alternatives (Lazer 2011; Cranmer and Desmarais 2016), and recent advancements in inferential network analysis have improved our ability to understand these complex systems (Cranmer et al. 2017). Notably, adoption of the exponential random graph model (ERGM) and its extensions, which allow for statistical inference explicitly on network generating processes, presents us with a way to test relational theories in a theoretically and statistically valid manner (Cranmer and Desmarais 2011; Lusher, Koskinen and Robins 2013). At the present, the ERGM family of models has been used to study a variety of political phenomenon.<sup>1</sup>

ERGMs are highly flexible in the features that can be specified as network generating processes, and by extension in the types of networks that can be modeled, but contemporary applications of ERGMs have been largely limited to systems containing relations of a single type. However, political systems often are not as cleanly self-contained as we imply when we treat them as monoplex networks, defined as those with only a single type of tie. Across various commonly studied political systems, researchers have been calling for recognition that there are meaningful interdependencies between them, or to treat these systems as subcomponents of larger processes (e.g. Nye and Keohane 1971; Keck and Sikkink 1998; Keohane 2009; Alter and Meunier 2009). For example, in studies of organized conflict, commonly identified mechanisms driving conflict initiation such as diversionary incentives (Downs and Rocke 1994; Tir and Jasinski 2008) often imply that different types of belligerent behavior within the global conflict system can be substitutes or complements of each other (Starr 1994). Another example can be found within the broad set of related research agendas on policy-making at different levels of political organization. In these policy networks, information transmission between political actors occurs through a variety of channels (Leifeld and Schneider 2012; Heaney 2014; Montgomery and Nyhan 2017), and tend to have complementary effects on whether influence is successful (Böhmelt 2010).

Complex interdependent systems such as the the global conflict system can be modeled as multilayer networks, a generalized framework for representing networks that are multi-

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<sup>1</sup>Examples include alliance ties (Cranmer, Desmarais and Kirkland 2012; Cranmer, Desmarais and Menninga 2012), political communication (Song 2014), international conflict (Cranmer and Desmarais 2011), lobby group influence (Heaney 2014), membership nominations (Leifeld and Fisher 2017), political contributions (Heaney and Leifeld 2017), immigration (Windzio 2018), economic sanctions (Cranmer, Heinrich and Desmarais 2014), and global governance (Hollway and Koskinen 2016).

modal, multiplex, temporally dynamic, or characterized by any combination of these complex features (Kivelä et al. 2014; Boccaletti et al. 2014). This approach opens important avenues of research, but at the present, the development of methods for studying multilayer networks is a relatively nascent endeavor. In particular, ERGM-based inferential network analysis is highly adaptable to multilayer networks, but aside from methodological illustrations using a specific subtype of multilayer networks (e.g. Wang et al. 2013, 2016), fully implemented multilayer applications of ERGMs that model interdependence across subsystems remain limited, especially in the study of politics (Hollway and Koskinen 2016).

This paper facilitates better understanding of inferential analysis of multilayer networks as an approach for studying complex interdependent systems comprising multiple relational contexts. I draw on developments in network science (Kivelä et al. 2014; Boccaletti et al. 2014), in particular the multilayer network as a generalized framework for representing networks comprising differentiated nodes and ties, to improve researchers' ability to better model social and political phenomena as they are intuitively understood. I describe an approach to conducting statistical inference on multilayer networks using the ERGM. There have been previous work on statistical inference of multilayer networks (Wang et al. 2013, 2016), but these applications focus on a specific subtype of multilayer networks. My work builds on these studies by describing the approach in terms of the generalized multilayer framework so as to encompass a wider range of social and political phenomena. Finally, this paper has a companion R package, `multilayer.ergm`, which facilitates the use of the ERGM with multilayer networks.<sup>2</sup>

The multilayer network approach to modeling political systems presents researchers with a tool to more carefully account for the complexity present in political phenomena. In addition to the applications I present in this paper, this approach is applicable to a wide range of topics in political science, including international regimes (Alter and Meunier 2009), legislatures (Montgomery and Nyhan 2017), and political campaigns (Heaney and Leifeld 2017). As a network-based approach, it facilitates a systems perspective to understanding politics without sacrificing the granularity that unit-level analysis affords. In the study of international relations, in particular, researchers have consistently presented the global system as one that features qualitatively distinct actors interacting with one another across a multitude of contexts that vary along many dimensions (Alter and Meunier 2009; Keck and Sikkink 1998). As I demonstrate in this paper, the multilayer network approach excels at these applications.

The remainder of the paper is organized into two sections. First, using conflict in the

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<sup>2</sup>A more detailed introduction of the `multilayer.ergm` package, along with replication code, is included in the appendix.

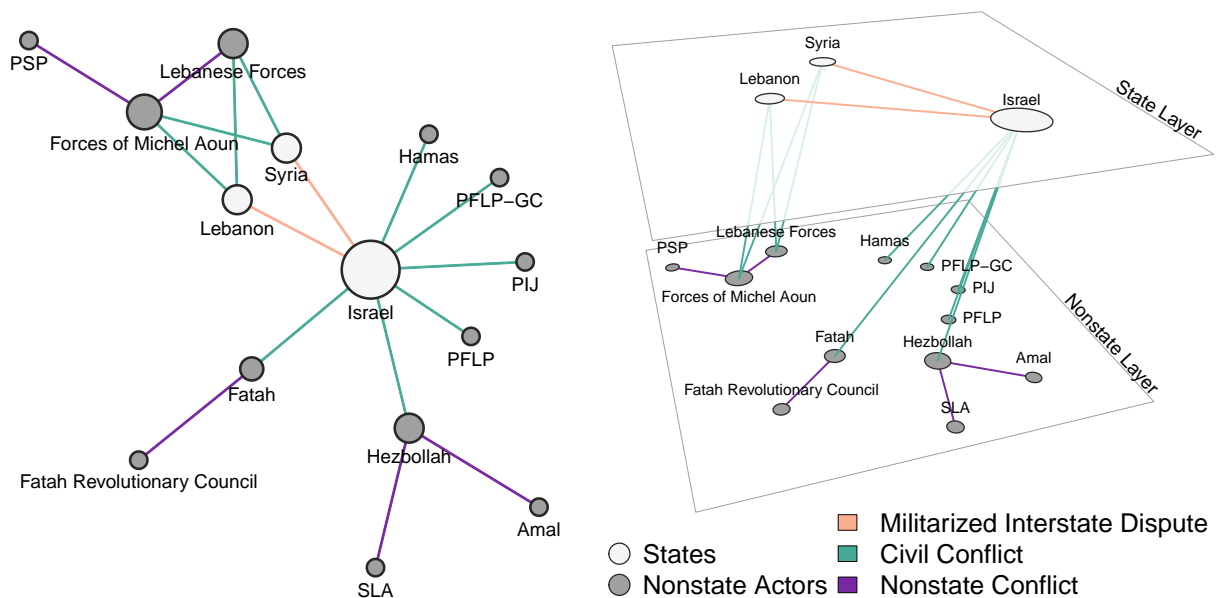
post-Cold War Levant as a running example, I describe the process of modeling systems characterized by interdependence that span multiple relational contexts using the ERGM, including how to formally represent these systems as multilayer networks and how to specify appropriate model terms. Second, I describe two types of multilayer networks, the multiplex and the node-colored networks, that are useful for representing a range of different social and political systems. I illustrate each type of multilayer network using a substantive application. For the multiplex network, I use a policy communication network with two types of communication ties; for the node-colored network, I use a global conflict network with three types of conflict ties. Using these applications, I demonstrate that the multilayer network approach produces models that better fit observed reality, and allows researchers to test more complex relational theories.

## 2 Statistical Inference for Multilayer Networks

Examples of the complex systems introduced above are all characterized by the existence of more than one type of relation existing between one or more groups of actors. These systems can be represented as networks, and as such, modeled using the ERGM. However, as these networks are marked by complexities not present in monoplex networks (i.e. those with a single type of tie), certain extensions are required. In this section, I discuss how multilayer network representations of complex systems afford inferential leverage over monoplex network representations, and show how this can be done through extensions to the monoplex application of ERGMs.

To more clearly illustrate the concepts I am presenting, throughout this section I use a small data set as my running example. Specifically, I use the conflict network between states and nonstate actors in the Levant during the post-Cold War period (1989-1995). This set of conflicts is a subset of the global conflict system which I use as one of my two applications later in the paper. In this system there are three states, twelve nonstate actors, and seventeen conflict ties of three different types: militarized interstate disputes, civil conflicts, and nonstate conflicts. Figure 1 contains two visualization of this network, with the traditional network plot on the left, and the multilayer network representation on the right. The multilayer representation groups nodes by their type into different layers such that the structure of the multilayer network is more easily discerned.

Figure 1: Levantine Conflict Network, Post-Cold War Period



## 2.1 Modeling Complex Systems Comprising Multiple Relational Contexts

The standard statistical modeling approach in conflict studies, network-based or otherwise, is to examine one set of conflict processes as the outcome variable of interest, while potentially specifying other types of conflicts as exogenous model covariates (e.g. Gleditsch, Salehyan and Schultz 2008). In terms of the Levantine conflict network, this means studying, for example, what led to Israel's conflicts with Lebanon and Syria. However, as is apparent from Figure 1, conflict in the region is not limited to these two incidents. Contemporaneous with these militarized interstate disputes (MIDs), all three states are engaged in conflicts with nonstate actors, some of whom are also in conflict with other nonstate actors. Isolated examination of these conflicts alone, even controlling for the presence of contemporaneous civil conflicts, misses interdependence that can exist across these processes.

To illustrate the advantage of simultaneously modeling different types of conflict processes, I discuss a set of models for explaining conflict in the Levant that differ in how they model interdependence in the network. To provide context, I first outline the events that led to the Israeli attack on Lebanon in July of 1993 (i.e. the reason for the tie between the two states on the Levantine conflict network). The direct precursor to the 1993 Israeli invasion into Lebanon can be traced to Israel's establishment of a security buffer zone in South Lebanon in 1982 (Luft 2000). The security zone was successful in driving Palestinian

Liberation Organization (PLO) member groups out of the region, but the hostile occupation resulted in the birth of the Hezbollah, which began as a movement to expel Israeli occupiers (Jaber 1997). Conflict persisted between Israel and Hezbollah through the early 1990s. In 1993, after a particularly intense period of Hezbollah aggression, Israel responded by launching a week-long attack into Lebanon.

The highly interdependent nature of the Levantine conflict network should be apparent from this account. To follow convention in the conflict studies literature and specify a model where MIDs are outcomes on interstate dyads and civil and nonstate conflicts are exogenous predictors requires the researcher to be satisfied with claiming that Hezbollah's decision to attack Israel was motivated by some reason completely unrelated to Israel's actions in Lebanon. This is an indefensible position given the wide recognition that the Hezbollah as a militant organization arose primarily in response to the Israeli occupation (Jaber 1997). This is but one example from the region, with similar types of cross-type interdependence manifesting throughout this conflict network.

It is possible to account for interdependence within the entire conflict system by moving to a joint examination of the three different conflict processes without differentiating between them. In the network analysis context, this is to model the system as a monoplex network. This approach can yield certain insights, as different types of conflicts share similar properties (Cunningham and Lemke 2013). From the traditional network plot in Figure 1, it is apparent the joint-but-monoplex network possesses features that are generally characteristic of conflict networks, such as high asymmetry in the extent to which conflicts cluster around certain actors, and the apparent lack of triadic closure, both of which are features of the global MID network (Cranmer and Desmarais 2011).

However, this is a suboptimal approach. While different types of conflicts reasonably share certain characteristics, there is heterogeneity between strategic considerations of states and nonstate actors facing conflict. This results in nuanced mechanisms across different relational contexts that cannot be captured by models that do not differentiate between different types of actors and conflict processes. Consider for example the most localized form of a conflict cluster, where one actor is simultaneously in conflict with two other actors. In such a scenario, strategic considerations for a state facing two other states (e.g. Israel versus Lebanon and Syria) is likely different than when it faces two nonstate actors (e.g. Israel versus PLO member groups), even after accounting for the usual set of model covariates. For one, despite weakening Westphalian norms, the international community still treats conflict between states differently from state-sanctioned violence within national borders. Because nonstate actors are not embedded in the interstate system, they are both less protected and less constrained. These differences are likely to result in conflict clustering to be driven

by different mechanisms. For example, interstate conflict clustering is likely to result from multiparty sanctioning efforts while civil conflict clustering is likely driven by the lack of international pressure on a state's internal behavior.

### 2.1.1 Presenting Complex Relational Systems as Multilayer Networks

In order to jointly examine these conflict subsystems while differentiating between them so as to be able to identify different cross-type conflict processes, we can utilize the multilayer network, a generalized framework that accounts for networks characterized by differentiated nodes and differentiated ties (Kivela et al. 2014; Boccaletti et al. 2014). The basic organizing principle of multilayer networks are network *layers*, which group nodes into disjoint subsets, with each layer containing one type of node. These layers and their associated node types are defined by the combination of one or more nodal attributes up to any number  $d$ . As presented in Figure 1, the Levantine conflict network has layers defined by one nodal attribute, statehood, which partitions the node set into state and non-state actor subsets. Generally, layers can be defined as a  $d$ -tuple, with each element in the tuple varying along its corresponding attribute, meaning that the set of layers comprises every combination of possible values across all relevant nodal attributes. A multilayer network with  $d$  relevant attributes each with  $k$  levels, has the following layer set where the superscript indexes the attribute and the subscript indexes the attribute level,  $\{(attr_1^1, attr_1^2, \dots, attr_1^d), (attr_1^1, attr_1^2, \dots, attr_2^d), \dots, (attr_k^1, attr_k^2, \dots, attr_k^d)\}$ .

These network layers are used to define different types of ties in the multilayer network. Specifically, tie types are identified by the combination of types of nodes they are incident to, which can be alternatively referred to by their incident layers. The two layers in the Levantine conflict network yields three types of ties, the militarized interstate dispute incident to two state layers; the nonstate conflict incident to two nonstate layers; and the civil conflict incident to one of each layer. By convention, ties incident to the same layer are referred to as intralayer ties and those incident to different layers are referred to as interlayer ties (Kivela et al. 2014).

Represented as a multilayer network, complex interdependent systems such as conflict in post-Cold War Levant can be approached using the variety of methods developed for network analysis. However, as most of these methods were initially developed for monoplex networks, extensions are required so they can account for the complex features of multilayer networks. As my purpose is to examine the generative process that underlies these systems, I focus on extensions to the exponential random graph model.



## 2.2 Multilayer Extensions to the Exponential Random Graph Model

Exponential random graph models (ERGM) are a class of statistical models that allows for inference on factors contributing to the generative process underlying an observed network, including exogenous node and dyad attributes, and network dependence effects at the dyadic-level and beyond. For example, a “pile-on” effect, whereby ties beget more ties, is a network effect that can manifest in observed conflict clusters (Cranmer and Desmarais 2011). Similar to a standard regression model, an ERGM is specified with a set of factors hypothesized to have contributed to the observed outcomes, in this case the set of ties on the network, and results from a fitted ERGM are the estimated sizes of these effects.

In an ERGM, the probability of observing a network  $\mathbf{Y} = \{Y_{ij}\}$ , where  $Y_{ij} = 1$  indicates a tie between nodes  $i$  and  $j$ , is specified as

$$Pr(\mathbf{Y}, \boldsymbol{\theta}) = \kappa^{-1} \exp\{\boldsymbol{\theta}'\mathbf{x}(\mathbf{Y})\}, \quad (1)$$

where  $\boldsymbol{\theta}$  is a vector of parameters and  $\mathbf{x}$  is a vector function that yields a vector of observed network statistics computed on  $\mathbf{Y}$ .  $\kappa = \sum_{\mathbf{Y}^* \in \mathcal{Y}} \exp\{\boldsymbol{\theta}'\mathbf{x}(\mathbf{Y}^*)\}$ , where  $\mathcal{Y}$  is the set of all networks defined on the node set of  $Y$ , is the normalizing constant that makes the equation a proper probability distribution.

Through the ERGM, researchers can determine whether hypothesized behavioral tendencies beyond node and dyad characteristics systematically contribute to observed relational patterns within a network. This is important because ties in political networks are rarely the result of only node activity and dyadic interaction. Instead, processes operating at the hyperdyadic level such as clustering or transitivity likely play important roles in tie formation among nodes (Cranmer and Desmarais 2016). These network effects, along with exogenous node and dyad covariates, which together form the generative model for the observed network, are specified as terms in the vector  $\mathbf{x}(\mathbf{Y})$ , usually in the form of counts of local network structures.

ERGMs are highly flexible in what is a permissible specification, limited only by the requirement that  $\mathbf{x}$  is finite when evaluated over any binary network (Desmarais and Cranmer 2012). Multilayer networks and appropriately specified statistics satisfy this criterion, but despite this flexibility,  $\mathbf{Y}$  is usually treated as a monoplex network, in turn constraining the statistics specified in  $\mathbf{x}$ . The key to extending the ERGM to multilayer networks lies in adapting  $\mathbf{Y}$  and  $\mathbf{x}$  in ways that account for properties of the multilayer network outlined above. This requires extensions to the matrix representation of  $\mathbf{Y}$  as a block matrix, which provides a reasonably straightforward way of incorporating layers into local network structures used as network models terms. I discuss these in turn below.

### 2.2.1 Data Structure

Networks are commonly represented using an adjacency matrix  $\mathbf{Y} = \{Y_{ij}\}$  where  $Y_{ij}$  indicates the presence of a tie between nodes  $i$  and  $j$ . For example, the militarized interstate dispute network with three nodes {Syria, Lebanon, Israel} indexed from 1 to 3, and two undirected ties {{Syria, Israel}, {Lebanon, Israel}} can be represented as

$$\mathbf{Y} = \begin{bmatrix} Y_{1,1} = 0 & Y_{1,2} = 0 & Y_{1,3} = 1 \\ Y_{2,1} = 0 & Y_{2,2} = 0 & Y_{2,3} = 1 \\ Y_{3,1} = 1 & Y_{3,2} = 1 & Y_{3,3} = 0 \end{bmatrix}. \quad (2)$$

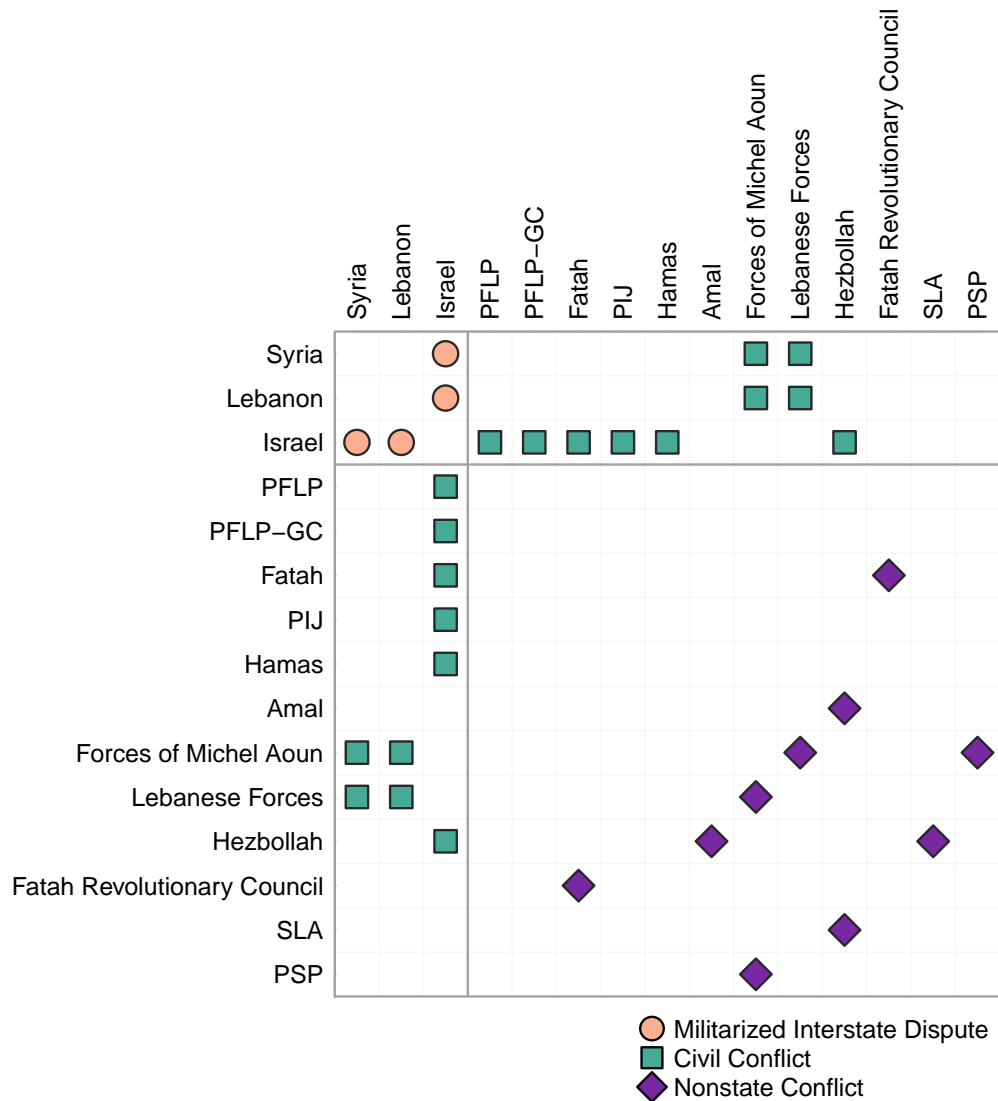
For the purposes of statistical inference of multilayer networks via the ERGM, the set of ties on a multilayer network with  $\ell$  layers can be grouped by type by partitioning the matrix into  $\ell \times \ell$  blocks:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}^{1,1} & \mathbf{Y}^{1,2} & \dots & \mathbf{Y}^{1,\ell} \\ \mathbf{Y}^{2,1} & \mathbf{Y}^{2,2} & \dots & \mathbf{Y}^{2,\ell} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}^{\ell,1} & \mathbf{Y}^{\ell,2} & \dots & \mathbf{Y}^{\ell,\ell} \end{bmatrix}. \quad (3)$$

This procedure places each network layer, with its intralayer ties, into one of the  $\ell$  main diagonal blocks, with the off-diagonal blocks in turn containing interlayer ties. As noted, depending on the specific multilayer network structure, these off-diagonal blocks may be constrained in different ways, discussed in more detail later. The matrix from Equation 2 would therefore occupy the (1, 1) block of the full adjacency matrix of the Levantine conflict network, which is graphically illustrated in Figure 2. In addition to the militarized interstate dispute ties on the (1, 1) block, the (2, 2) block represents the nonstate conflict ties, and the off-diagonal blocks (1, 2) and (2, 1), which contain the same information because this network is undirected, represent the interlayer, civil conflict ties.

An additional step in constructing the multilayer adjacency matrix is to specify the constraints appropriate for the overall structure of the network. Applying sampling constraints to the adjacency matrix, usually in the form of disallowing ties to be formed between certain nodes, is a relevant consideration to all ERGM examinations of networks. In any network, contextual characteristics of the system can require that ties never be formed between certain nodes. For example, in a temporally-pooled examination of interstate relations, state entry and exit from the interstate system mean that there are states whose existence do not overlap, such as for Czechoslovakia (exit in 1993) and Palau (entry in 1994). Aside from these contextually idiosyncratic requirements, certain types of multilayer networks require system-

Figure 2: Adjacency Matrix for Post-Cold War Levantine Conflict Network



atic constraints on their off-diagonal blocks. I discuss these in more detail in the application section.

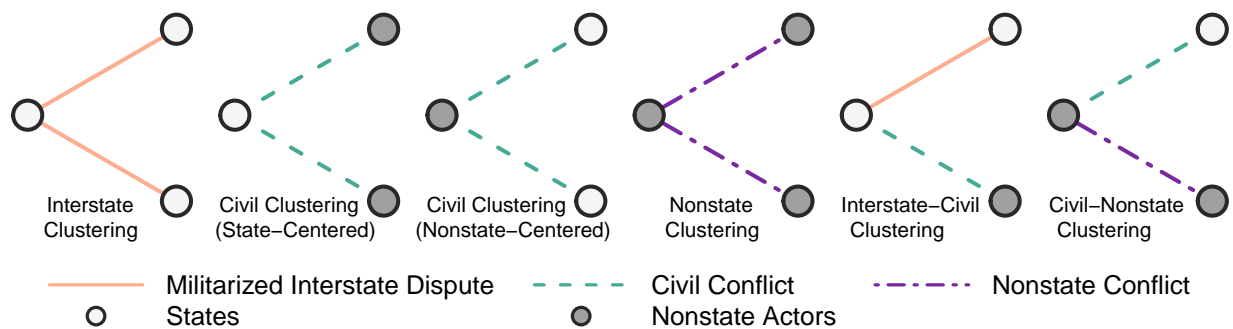
### 2.2.2 Local Network Structures in Multilayer Networks

The main contribution of the multilayer extension to ERGMs is in the network dependence structures that can be used to account for phenomenon across different types of ties. These cross-layer dependence structures should be specified by the researcher based on appropriate theoretical expectations. In the application section I demonstrate the use of various cross-layer dependence structures, but a full discussion of possible multilayer terms is beyond the

scope of this paper. For a detailed description of a broad set of these structures, readers are referred to Wang et al. (2013, 2016). Here, to illustrate the basic construction of a cross-layer network structure, I discuss how clustering structures on a monoplex version of the Levantine conflict network differ from that on multilayer version of the same network.

On a monoplex network, the lowest form of tie-clustering is represented by a two-star, defined as two ties that share a node (Robins et al. 2007). The corresponding network statistic is a count of two-stars on the network. On the multilayer network with three types of conflict ties, there are six different types of two-star clusters, as shown in Figure 3. In order to obtain the size of the desired subset, ties are only counted when they belong to the appropriate blocks. For example, to obtain a count of two-star comprised of a militarized interstate dispute and a civil war, instead of computing the statistic over the entire network, only the  $\mathbf{Y}^{1,1}$  and  $\mathbf{Y}^{1,2}$  blocks from Figure 2 are used. This logic of identifying and limiting counted ties by matrix block remains the same for all cross-layer network structures regardless of their complexity.

Figure 3: Six Different Types of Two-Star Conflict Clustering



After a complex system has been modeled as a multilayer network and the network represented as an adjacency matrix, statistical inference on the generative process of the system is an exercise in specifying the theoretically appropriate model terms and sampling constraints for the ERGM.

### 3 Applications

In this section, I outline two substantive applications that are aimed at demonstrating the use of ERGMs with multilayer networks. These applications are selected to show how the general multilayer framework can be structured to encompass different types of multilayer networks to represent a wide range of commonly observed relational systems. Specifically,

I describe the *multiplex network*, which captures systems with multiple types of relations among a single type of actor; and the *node-colored network*, which captures systems with different types of actors, where ties differ by the combination of interacting actors.

For each application, I discuss the characteristics of the system and introduce the type of multilayer network it fits. I describe characteristics of these multilayer networks including whether actors exist on multiple layers and how the interlayer ties are structured. To make these examples more tractable, I limit the networks discussed to those with only two layers. However, the framework introduced here readily allows multilayer networks with greater than two layers.

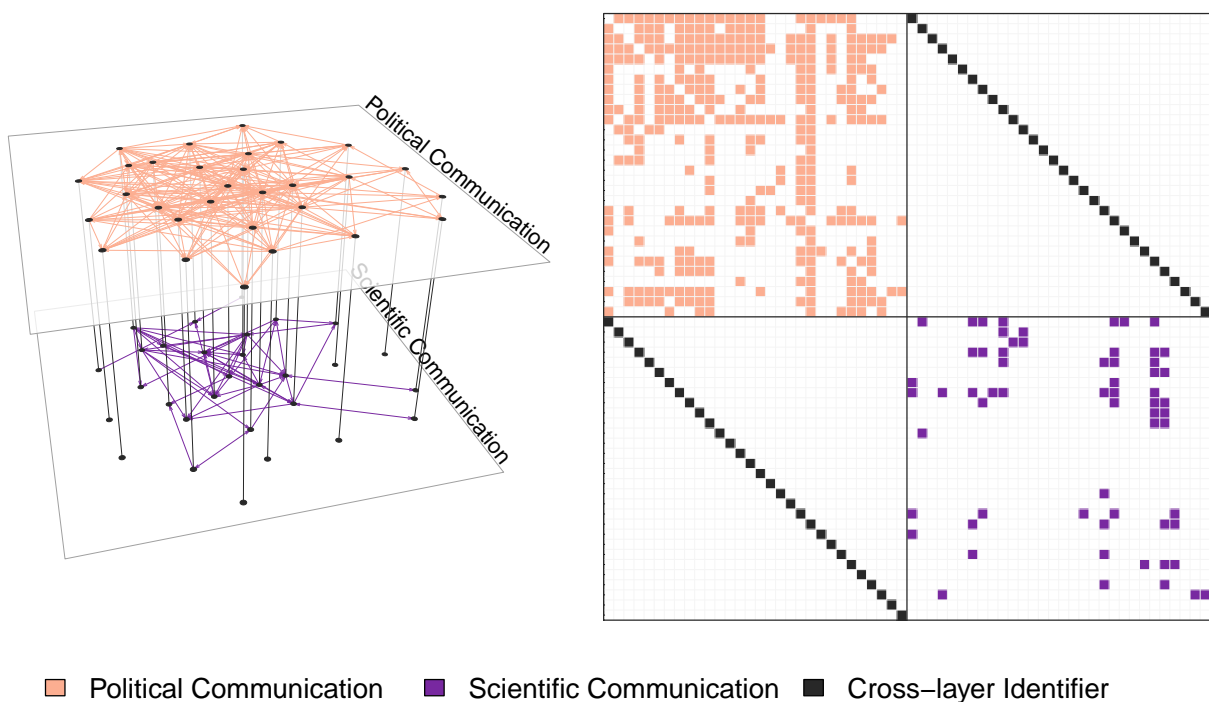
### 3.1 Multiplex Networks: The 1980 ChemG Policy Communication Network

To demonstrate the use of multilayer networks to model multiple relations among a set of actors of a single type, I draw upon a study conducted by Leifeld and Schneider (2012) on information exchange in policy networks. In this study, Leifeld and Schneider examined the patterns of two types of information exchange, political and scientific, among thirty actors who were relevant to the 1980 German legislation on toxic chemicals regulation (ChemG). This set of observed actors and their interactions can be represented as a directed *multiplex* network with two layers, where ties indicate flow of one of the two kinds of information. Figure 4 contains the graphical and matrix representations of this network.

#### 3.1.1 Multiplex Networks

Multiplex networks are those where different types of ties exist between the same set of real-world actors. Each of these tie types exists on a different network layer. This is a commonly observed network structure, as actors within a system are rarely restricted to only a single type of relation. A policy network where actors share different types of information using different communication channels, or an interstate network of both conflict and trade ties are both examples of multiplex networks. The multiplex network requires constraints placed on the off-diagonal blocks of the adjacency matrix. By definition there is only one set of real-world actors represented by multiple nodes on the network, and interlayer ties are used to couple nodes representing the same actors across layers. These self-coupling interlayer ties are almost always treated as fixed. Specifically, when the relational data is represented as an adjacency matrix, the off-diagonal blocks of the matrix are identity matrices. This data structure is presented in Figure 4 using the German ChemG policy network data from Leifeld and Schneider (2012). The identity matrices are considered when calculating network

Figure 4: Multiplex Representation of ChemG Policy Network



statistics but are not modeled as outcomes. Existing approaches to statistical inference on multiplex networks (e.g. Leifeld and Schneider 2012; Heaney 2014; Song 2014) often treat one layer of the network as the “outcome” network that is allowed to vary, while fixing the other layers as exogenous predictors. Under certain conditions this approach can be reasonable, but elsewhere the underlying assumptions are violated. In these cases, where the two networks are mutually reinforcing, the inferential result will be subject to simultaneity bias on the estimated parameters.

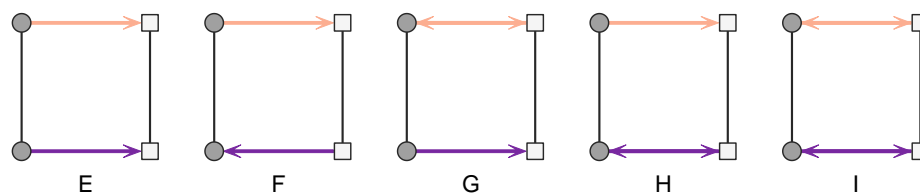
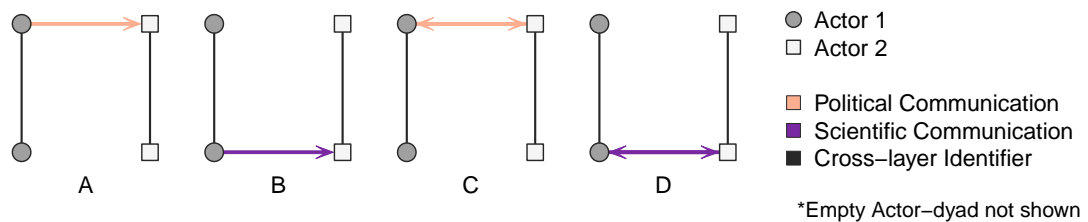
### 3.1.2 Extensions to Leifeld and Schneider (2012)

The question Leifeld and Schneider posed is what drives tie formation on these two policy communication networks. They base their argument on a transaction costs approach. As they argue, policy actors send information to each other as attempts at influencing their policy positions. Since establishing contact is costly, actors are more likely to utilize existing channels of communication as opportunity structures to attempt influence at a relatively low cost. Based on this logic, they derive a number of expectations. Here, I limit my multiplex extension to two of these hypotheses, as this is intended as an illustrative exercise.

First, Leifeld and Schneider argue that information exchange is likely to be recipi-

cated. To test this, they include a term for reciprocal tie formation in both of their political and scientific communication models. For both models, the term was statistically significant with a positive sign, indicating that reciprocal tie formation is as hypothesized a part of the underlying generative process for the observed networks. These monoplex, within-layer examinations, however, cannot account for the potential for more complex forms of reciprocity to occur across network layers. Consider that within a given actor-dyad, where two types of ties are possible for both directions, there are ten different observable combinations of information flow. Figure 5 outlines these combinations with the exception of no information exchange. Of these ten possible information-sharing scenarios between two actors, four include some form of reciprocal tie formation that cannot be modeled without considering cross-layer dependence. For example, when structure  $F$  is included as a model term, it captures the tendency for one kind of information exchange to be reciprocated through other means, which is expected in policy networks with actor specialization. The other three structures ( $G$ ,  $H$ , and  $I$ ) capture complex forms of conditional reciprocity. In the multiplex model I include these cross-layer dependence terms.

Figure 5: Census of Dyad Configurations in a Directed Duplex Network



Second, the authors argue that actors who exchange political information are more likely to exchange scientific information and vice versa. For this kind of mutual reinforcement between political and scientific communication, the authors modeled the two networks separately while including the other network as an exogenous predictor. However, specifying edge or node covariates that are simultaneous with the outcome variable in an ERGM as exoge-

nous will result in the same type of simultaneity bias as when endogenous covariates are specified as exogenous in classical regression models. The multiplex approach allows for the inclusion of a dependence term that directly measures cross-layer reinforced tie formation. This term is visualized as structure  $E$  in Figure 5, which I include in my multiplex extension.

In this extension, I fit two models of the ChemG multiplex network. The first is a layer independence model that includes both network layers but no cross-layer dependence terms. The model specification here follows exactly those found in Leifeld and Schneider (2012). The second is a cross-layer dependence model that, in addition to the within-layer terms from the layer independence model, adds cross-layer dependence terms described above. The full reciprocity term captured by structure  $I$  is omitted to avoid multicollinearity.

### 3.1.3 Results

Results from this application are presented in Table 1. The first column contains the two communication networks jointly modeled but without cross-layer dependence terms. To examine this model's ability to capture characteristics of the multilayer network, I conducted a goodness-of-fit test. This is done by simulating 1000 networks using the layer independence model, and comparing the distribution of relevant network statistics to the observed statistics. The results summarized in Table 2 indicate that the layer independence model does not generate the observed cross-layer structures well. Evidently, cross-layer dependence is a feature of the overall multiplex communication network. Table 2 also shows goodness-of-fit results from the cross-layer dependence model, which are a near perfect match as is expected from a converged model that directly models these structures.

The second column of Table 1 contains the cross-layer dependence model. Of the four modeled cross-level dependence terms, two are statistically significant at the 0.05 level. First, the cross-layer reinforcement term is positive and significant, with an estimated log-odds of 3.48. This means that, conditional on all other modeled effects, an actor already sending one type of information to a partner is 32 times as likely to send the other kind of information to the same partner than to someone else. This result supports Leifeld and Schneider's costly influence argument in that if an actor is going to use its resources to attempt influence, it will do so where opportunity structures already exist.

The other statistically significant term is the effect captured by the the presence of reciprocated scientific communication and unreciprocated political communication between a pair of actors (structure  $H$  in Figure 5). The negative sign (log-odds of  $-3.45$ ) on this term indicates that despite the tendency for reinforcement and reciprocity to facilitate information exchange in this network, these effects are attenuated when the pair of policy actors are reciprocally sharing information that is at least in part scientific. For example, the condi-



Table 1: Multiplex Extension of Leifeld and Schneider (2012)

Term	Layer Independence		Cross-layer Dependence	
	Estimate	s.e.	Estimate	s.e.
<u>Political Communication</u>				
Arcs	-5.06*	1.00	-5.21*	1.01
Reciprocity	0.81*	0.25	0.50	0.27
GWESP(0.1)	2.43*	0.85	2.53*	0.86
GDSP(0.1)	-0.13*	0.05	-0.12*	0.04
Government Receiver	0.60*	0.19	0.65*	0.19
Scientific Sender	0.07	0.22	0.12	0.22
Interest Group Interaction	1.05*	0.29	1.03*	0.30
Committees Arc	0.30*	0.05	0.31*	0.05
Influence Arc	0.93*	0.18	0.93*	0.18
Preferences Arc	0.11	0.07	0.12	0.07
Scientific Arc	2.88*	0.62		
<u>Scientific Communication</u>				
Arcs	-5.98*	0.79	-7.52*	1.24
Reciprocity	1.76*	0.52	7.68*	2.28
GWESP(0.1)	0.49*	0.23	0.57*	0.22
GDSP(0.1)	-0.17	0.10	-0.19	0.09
Government Receiver	0.37	0.32	0.37	0.31
Scientific Sender	1.54*	0.37	1.74*	0.36
Interest Group Interaction	1.22*	0.60	1.05	0.60
Committees Arc	0.14*	0.06	0.09	0.05
Influence Arc	0.32	0.31	0.22	0.30
Preferences Arc	-0.02	0.12	-0.09	0.11
Political Arc	2.87*	0.61		
<u>Cross-layer Dependence</u>				
Reinforcement			3.48*	1.14
Reciprocity			1.71	0.99
Scientific Arc Political Reciprocity			0.20	0.94
Political Arc Scientific Reciprocity			-3.45*	1.18

\*  $p < 0.05$ .

Both models were fit in R using the `multilayer.ergm` package.

tional effect of cross-layer reinforcement on sharing political information is reduced to just 5.4 times as likely (down from 32 times) when the pair of actors are already reciprocating scientific information. One interpretation of this finding is that there are actors who use policy communication for meaningful cooperation as opposed to for attempted influence. In these cases, cross-layer alignment become less likely because policy actors specialize, and without the incentive to attempt influence, sending information outside of one's specialization

Table 2: Goodness-of-Fit for Cross-layer Structures

Cross-layer Term	Observed	Layer Indep.		Cross-layer Dep.	
		Sim. Mean	s.d.	Sim. Mean	s.d.
Reinforcement	60	44.9	7.3	61.5	9.3
Reciprocity	55	40.2	7.2	56.5	9.1
Scientific Arc Political Reciprocity	53	33.3	6.8	54.5	9.0
Political Arc Scientific Reciprocity	20	17.8	6.1	21.0	7.2

Results based on 1000 simulations for each model.

All simulations were conducted in R.

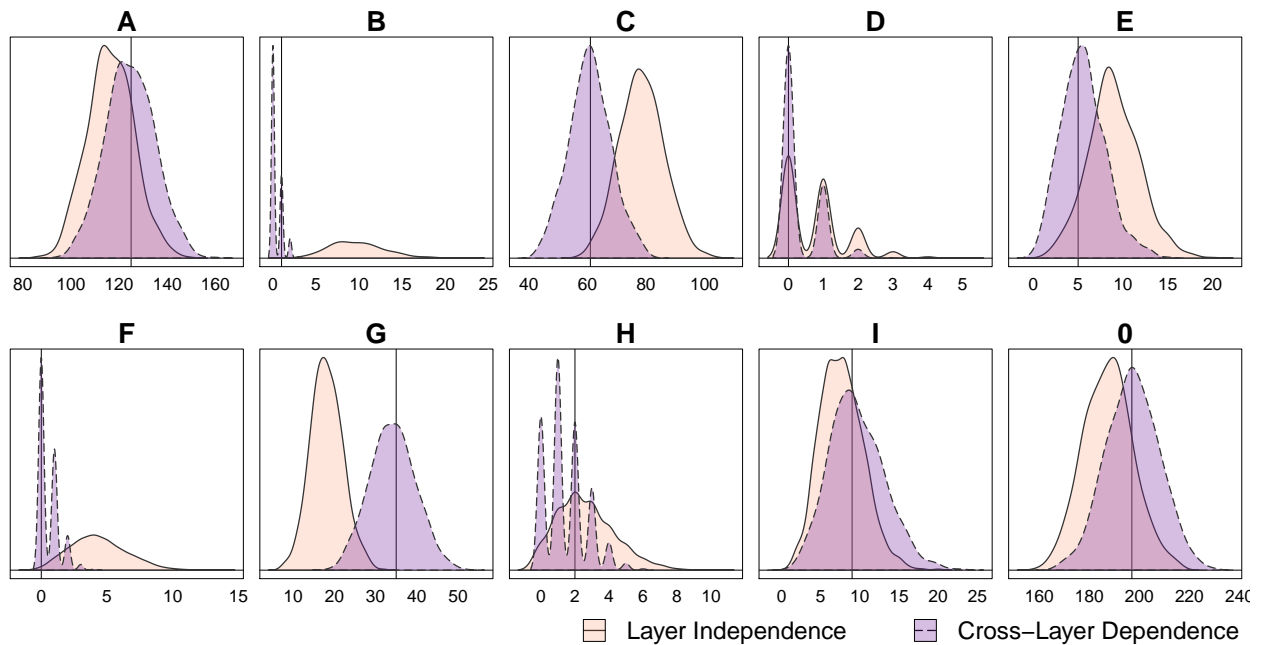
becomes superfluous and not done. These results indicate that while the general transaction cost framework of understanding policy communication still hold, there are additional dynamics governing information exchange that are undetectable using layer independence approaches.

In addition to expanding the types of substantive relational theories that can be examined, inclusion of cross-layer dependence structures can improve model fit to the observed data. To demonstrate this, I simulate 1000 networks from each model and examine the resulting distribution of the cross-layer dyad census presented in Figure 5. The results from these simulations are visualized in Figure 6. In each subfigure, the simulated distribution of counts of the given cross-layer dyad structure for the layer independence and the cross-layer dependence models are respectively shown with darker (dashed outline) and lighter (solid outline) ink. The vertical black line marks the observed value of the given structure. These results indicate that the cross-layer dependence model outperforms the layer independence model in terms of fitting the distribution of types of interactions between pairs of actors.

### 3.2 Node-colored Networks: The Global Conflict Network

For my second application, I model the global conflict system in the post-Cold War period as an undirected multilayer network. In this application, I focus on the tendency for conflicts to cluster within and across different conflict types, which I discussed above using the Levantine conflict network. As previously noted, researchers are increasingly recognizing that the collective body of organized political conflict, which includes interstate conflict, intrastate conflict, nonstate conflict, and one-sided violence against civilians, are interdependent events within the same system (Allansson, Melander and Themnér 2017; Gleditsch, Salehyan and Schultz 2008; Starr 1994). In systems such as these, where we expect strategic interdependence to manifest across subsystems (Tir and Jasinski 2008; Martinez Machain and Rosenberg 2018), a monoplex approach requires assumptions about interlayer dependence often not supported

Figure 6: Comparison of Model Fit for Dyad Census



by theory, potentially resulting in an incomplete understanding of the set of mechanisms underlying tie generation and in poorly fitting models. My present application is intended to illustrate the advantages of the multilayer approach to modeling these type of systems as *node-colored* networks.

### 3.2.1 Node-colored Networks

A node-colored network comprises different types of actors, represented by “colors” that function as categorical labels for actor type (Kivelä et al. 2014). Nodes are grouped by their colors into different layers, which means that intralayer ties are those that exist between nodes of the same color, and interlayer ties between nodes with different colors. Because actors in node-colored networks never exist on more than one layer, interlayer ties are not used for coupling nodes across layers as they are in multiplex networks. Instead, they represent ties between two different real-world actors and are therefore substantively-meaningful relational outcomes that can be modeled. As such, whereas a multiplex network has as many types of ties as it does layers, a node-colored network with  $k$  colors has up to  $\binom{k}{2}$  types of ties. By this definition, bipartite networks are a specific case of node-colored networks where intralayer ties are assumed away. For example, political contributions modeled as a bipartite network assumes independence between different receivers and between different donors aside from shared cross-type partners (e.g. Heaney and Leifeld 2017). Unless a node-colored network is

truly bipartite, it is desirable to model it as a multilayer network by treating intralayer ties as outcomes.

Because the interlayer ties of node-colored networks are usually relational outcomes, systematic constraints are generally not placed on the off-diagonal blocks of node-colored networks. The exception to this is when the interlayer ties represent hierarchical affiliation ties (e.g. employment, citizenship) between nodes at in different layers (Wang et al. 2013). For example, a congressional member network and a congressional staffer network (Montgomery and Nyhan 2017) can be represented as a node-colored network with congressional members on one layer and their staffers on another. In these networks, if affiliations are generally stable and not of theoretical interest, the off-diagonal blocks may reasonably be fixed at their observed values.

### 3.2.2 Modeling Conflict Clustering across conflict Types

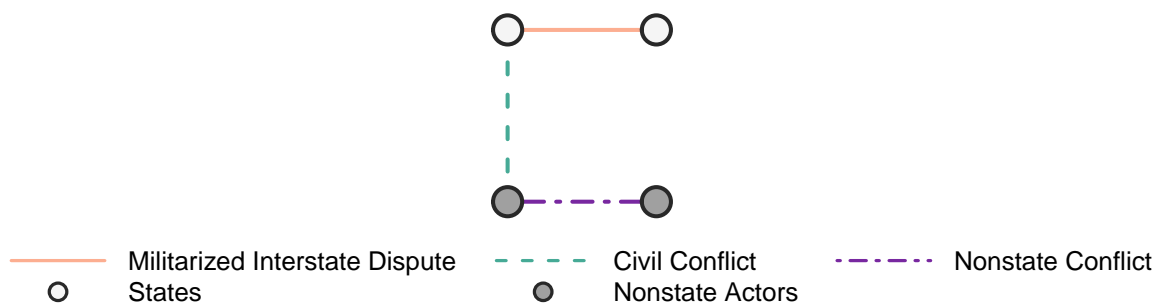
In this application I focus on the tendency for conflicts to cluster. I base my model on a study by Gleditsch, Salehyan and Schultz (2008), which examines the relationship between interstate conflict and the presence of civil wars in a country. More specifically, the authors examines militarized interstate dispute (MID) onset for all politically-relevant dyads during the period of 1948-2000 and find that the presence of civil conflict in at least one state in the dyad predicts MID onset. This relationship should manifest as clustering of interstate and civil conflicts in the system. In their model specification, Gleditsch, Salehyan, and Schultz include the presence of civil conflict in the dyad as an exogenous predictor to MID onset. Specifying one type of conflict as the outcome and the others as exogenous predictors is the current standard in modeling the relationship between different types of conflicts, using the ERGM or otherwise. As illustrated using the Levantine conflict network earlier, what this approach misses is that the presence of different types of intrastate conflict is highly likely to be endogenous to interstate conflict through both simultaneity and confounding.

My multilayer extension departs from Gleditsch, Salehyan and Schultz's (2008) study in a number of ways. First, using the multilayer approach, I model all potential conflicts in the global system (i.e. interstate, civil, and nonstate conflicts) as outcomes instead of focusing on politically-relevant interstate dyads. Second, to limit the methodological scope of my examination to something that is tractable for this paper, I focus on a single temporal cross-section of the global conflict system instead of a time-series. Specifically, I examine the period immediately following the end of the Cold War, from 1989 to 1995. This period was marked by a drastic change in the balance of power between states and nonstate actors in the periphery in favor of the latter (Kalyvas and Balcells 2010). The overall strategic environment at the time, ripe for potential conflict clustering across types, presents an apt opportunity for

illustrating the importance of modeling interdependence across different conflict processes.<sup>3</sup>

Similar to the ChemG application, I fit a layer independence and a cross-layer dependence model. The two model specifications are identical except for how dependence across different types of conflicts is modeled. Specifically, both models include within-layer star terms to capture the tendency for within-type conflict clustering, but where the layer independence model approaches cross-type clustering by specifying one type of conflict as an exogenous predictor of another, the cross-layer dependence model uses cross-layer dependence star terms. Further, the cross-layer dependence model also includes a three-way clustering term that captures the tendency for all three types of conflict to cluster at the same time, which is illustrated in Figure 7. This term cannot be modeled in a straightforward manner using conventional approaches. A discussion of the data used, network creation procedure, and modeling steps is included in the appendix.

Figure 7: Local Network Structure for Three-way Clustering: Cross-layer Three Path



### 3.2.3 Results

Table 3 contains results from this application. As noted, the models have identical specifications for within-layer terms. The substantive interpretation of these terms do not differ between models, and generally corroborate expectations from the conflict studies literature. Notably, all dyadic terms that are included in Gleditsch, Salehyan and Schultz (2008, Table 2) have the same estimated signs as the original study. The alternating  $k$ -star terms for all three types of conflicts are positively signed in both models, indicating tendency for

<sup>3</sup>While I focus on a single time period, the multilayer network framework is not limited to temporally static networks. To model temporal dynamics in a relational system, time series of networks can be represented as multiplex networks, where each network in the series is a network layer. The network in the time series can be a multilayer network, meaning that a time series of the global conflict network can be represented as a multiplex, node-colored network. In this case, a tie's type is defined not just by conflict, but conflict-time (e.g. MID in 1992).

Table 3: Models of the Post-Cold War Global Conflict Network

Term	Layer Independence		Cross-layer Dependence	
	Estimate	s.e.	Estimate	s.e.
<u>Militarized Interstate Dispute (MID)</u>				
Edges	-11.19*	0.93	-13.43*	1.23
Isolates	-2.20*	0.53	-3.16*	0.63
Alternating $k$ -star(2)	1.45*	0.29	1.59*	0.27
Democratic State	0.33	0.20	0.28	0.18
CINC	6.66*	2.65	4.79*	2.34
Democratic Dyad	-1.23*	0.45	-1.39*	0.43
Contiguity	2.93*	0.29	2.95*	0.28
Politically Relevant Dyad	2.00*	0.36	1.94*	0.35
CINC Ratio	-0.23*	0.07	-0.24*	0.06
State in Civil Conflict	0.61*	0.14		
<u>Civil Conflict</u>				
Edges	-8.21*	0.26	-8.38*	0.40
State-centered Alternating $k$ -star(2)	1.25*	0.13	1.26*	0.13
Democratic State	-0.22	0.14	-0.18	0.13
Transitional State	0.29*	0.11	0.34*	0.11
State in MID	0.93*	0.26		
Nonstate in Conflict	-0.45*	0.17		
<u>Nonstate Conflict</u>				
Edges	-15.94*	2.33	-13.84*	2.48
Isolates	-4.51*	1.18	-3.60*	1.25
2-star	-0.48	0.37	-0.49	0.40
Alternating $k$ -star(2)	4.27*	1.35	4.29*	1.43
Nonstate Actor in Civil Conflict	-0.10	0.12		
<u>Cross-layer Dependence</u>				
MID-Civil Conflict Alternating $k$ -star(1)			1.27*	0.43
Civil-Nonstate Conflict Alternating $k$ -star(1)			-1.12*	0.25
Three-way Clustering			0.19*	0.07

\*  $p < 0.05$ .

Both models were fit in R using the `multilayer.ergm` package.

within-type ties to cluster.<sup>4</sup>

Moving to the cross-conflict clustering terms from the cross-layer dependence model, note that the alternating  $k$ -star term for MID-civil conflict clustering is positive, indicating

<sup>4</sup>The alternating  $k$ -star is an extension to the two-star count that captures the tendency for ties to cluster, but attenuates this tendency as clustering increases (Snijders et al. 2006; Robins et al. 2007). In addition to its theoretical appeal, the alternating  $k$ -star terms improve model fit and reduce their tendency for degeneracy in the estimation procedure (Snijders et al. 2006). Both models include an additional two-star term for nonstate conflict ties. The inclusion of multiple  $k$ - or alternating  $k$ -star terms is sometimes required to obtain models with sufficiently good fit (Wang et al. 2013). In this case, both models have large, positive coefficients for their alternating  $k$ -star terms, with a negative but much smaller two-star coefficient. Jointly considered, the coefficient estimates on these two terms indicate strong tendency for clustering of nonstate conflict ties.

that conditional on involvement in a MID, the state is likely to engage in civil conflict, and vice versa. This result is in line with results from Gleditsch, Salehyan and Schultz (2008). On the other hand, the alternating  $k$ -star term for civil-nonstate conflict clustering is negative, meaning that nonstate actors tend to avoid entering into conflicts simultaneously with mixed opponent types. This finding is within theoretical expectations (Fjelde and Nilsson 2012), but previously has been shown only with indirect evidence.

The key theoretical contribution from the cross-layer dependence model comes from its three-way clustering term. The estimated coefficient for this term is positively signed, meaning that the odds of a conflict tie forming when it attaches to an existing cluster of the other two types of conflicts is higher than when it is concurrent with only one conflict. Evidence of this kind of interdependence spanning all three types of conflict ties indicates that when states make decisions about their conflict engagements with other states, the neighborhood they consider extends beyond the interstate dyad and its concurrent civil conflict ties, to whether their civil conflict partners are also engaged in conflicts of their own. This finding is intuitive, but prior to the multilayer network approach had not been demonstrated in a straightforward manner.

## 4 Conclusion

We understand political phenomena as systems of units interacting across multiple relational contexts. The increasing application of ERGMs to modeling these phenomena has yielded theoretical advancements across various areas in the study of politics, but contemporary network approaches still require assumptions about independence of ties across different relational contexts in a manner that does not comport with our understanding of politics. Despite these limitations, relatively little work has been done in adapting network-based modeling approaches for capturing these interdependencies.

In this paper, I illustrated a multilayer network approach that extends the ERGM to account for interdependence in complex systems comprising multiple relations. Through two substantive applications, I demonstrated that the multilayer approach to ERGMs affords us the opportunity to model interdependence across multiple relational contexts using cross-layer dependence terms and to examine these interdependencies as generative features of the system. In the ChemG policy communication application, I showed that reciprocity between actors is more complex than what can be inferred when focusing on only one type of information exchange. Instead, jointly modeling multiple communication networks can yield insights into the conditions under which pairs of actors reciprocate information. In my examination of the global conflict system, I showed that cross-layer dependence terms

can be used to model the tendency for different combinations of conflicts types to cluster. Results here suggest that when making strategic choices regarding conflict, states assess not only their and their opponent's immediate conflict engagements, but are attentive to the local conflict environment the dyad is situated in, including the extent to which their civil conflict partners are engaged in other conflicts. Just as importantly, I showed using simulation exercises that by accounting for existing cross-layer dependence, the cross-layer dependence models fit observed data better than models constrained be independent across layers.

My present argument for treating multiple systems previously examined independently of one another as components in a larger system comprising multiple relational contexts is an argument for more carefully considering the complexity present in political phenomenon. In addition to the applications presented in my paper, this is a consideration salient to many other commonly studied political systems, such as international regimes (Alter and Meunier 2009), legislatures (Montgomery and Nyhan 2017), and political campaigns (Heaney and Leifeld 2017). This argument is not new, but despite the ostensibly wide recognition for these complexities, methods for working with them were lacking. The multilayer extension to the ERGM presented here should afford researchers a useful tool in this endeavor.



## A Companion R package: `multilayer.ergm`

`multilayer.ergm` is an R package that extends the exponential random graph model to multilayer networks. It does so by extending functionalities of the `ergm` package to provide a way to count local network configurations, or network motifs, that span more than one network layer.

The package lives at its Github page (<https://github.com/tedhchen/multilayer.ergm>) and can be installed by entering the following into the R console. This requires you to have the `devtools` package installed. If you are using the Windows operating system, you will also need the standalone Rtools.<sup>5</sup> The argument `build_opts` can be set to default (i.e. remove the argument when running the function) if you do not require the package tutorial.

```
devtools::install_github("tedhchen/multilayer.ergm",  
                          build_opts = c("--no-resave-data", "--no-manual"))
```

The package tutorial can be accessed by entering the following into the R console. The tutorial contains a brief introduction of how to create multilayer networks from multiplex networks and how to fit an ERGM to the multilayer network.

```
vignette("policy_multiplex")
```

The package is subject to further development. Version 0.1.6 ensures reproduction of results from this paper. For updates, please visit the package Github repository.

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<sup>5</sup><https://cran.r-project.org/bin/windows/Rtools/>

## B Global Conflict Network: Data and Methods

This section contains a description of the data and methods for the global conflict application.

### B.1 Data, Measures, and Network Creation

This study requires two sets of data. First, conflict data is needed to construct the multilayer network. For this, I used dyadic militarized interstate dispute data (Maoz 2005) for interstate conflict ties, and conflict data from the Uppsala Conflict Data Program (Allansson, Melander and Themnér 2017) for civil conflict ties and nonstate conflict ties. Second, data on node and dyad characteristics are required for the ERGM specification. I obtained this set of data from the replication files of Gleditsch, Salehyan and Schultz (2008).

Most of the raw data exist in either time-stamped or annual summary format. Because I model the post-Cold War years (1989–1995) as a single time period, the first step in processing the data is to aggregate them for the period. My aggregation algorithms are summarized in Table 4.

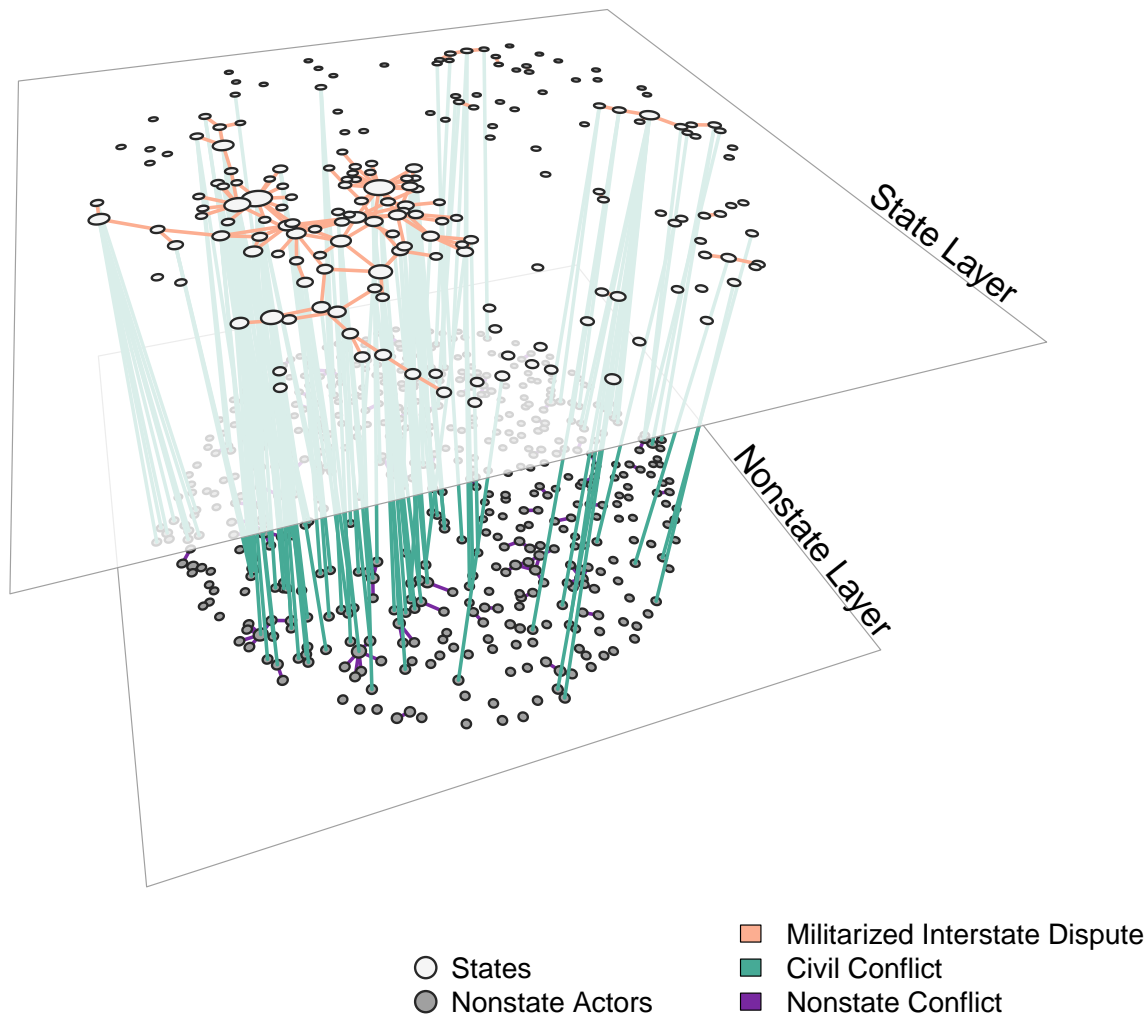
Table 4: Summary of Algorithm Used to Aggregate Data

Measures	Aggregation Algorithm
MID onset; civil conflict; nonstate conflict	Presence of at least one observation during period
Democracy*; joint democracy*	Held status for entire period
Transitional regime; political relevance; contiguity	Held status for at least one year during period.
Logged CINC; logged CINC ratio*	Median annual value during period

\*These measures are lagged by one year for the entire period.

Next, to create the global conflict network, I first defined the set of nodes. All states that were part of the interstate system during the period are included, with the exception of Samoa, for which several variables are missing with no appropriate substitute. Nonstate actors are included if they were observed to be involved in any kind of organized conflict up to five years before and after the period under examination (1984-2000), based on data from the Uppsala Conflict Data Program (Allansson, Melander and Themnér 2017). Specifically, I used version 17.1 of the Dyadic Dataset, the Non-State Conflict Dataset, and the One-sided Violence Dataset. As in the Levantine conflict network, these nodes are organized by their statehood into two layers, state and nonstate. These two layers are then respectively populated with 189 states and 544 nonstate actors. As a node-colored network, the two layers yield three sets of ties based on the conflict data described above. This network is visualized in Figure 8.

Figure 8: Global Conflict Network, Post-Cold War Period



## B.2 Modeling Steps

In this section, I describe the steps I take to arrive at my model of the global conflict network with cross-layer dependence. As in the ChemG application, I fit two ERGMs that differ by whether they include cross-layer dependence terms, and show that accounting for cross-layer dependence yields both substantive insights and a model that better fits the observed data. Whereas in the previous application I was able to use Leifeld and Schneider's (2012) model specification, extending it by adding cross-layer terms, the present application requires that I begin by finding a well-fitting model for each of the three sets of ties.

First, I specify the presence of civil conflict as an exogenous covariate for state nodes in

the MID submodel following Gleditsch, Salehyan and Schultz (2008). Specifically, engagement in civil conflict is specified as a nodal attribute that influences the likelihood of a state to form MID ties on the network. In network models, this is commonly referred to as an “activity” effect. Similar to the MID submodel, the submodels for civil and nonstate conflict networks include activity terms for whether nodes are presently engaged in the type of conflict not modeled as network ties. For additional exogenous node and dyad covariates, I draw on the set of variables from Gleditsch, Salehyan and Schultz (2008, Table 2, p. 491) as reference for the MID submodel. The submodels for the civil and nonstate conflict ties are sparse, as there is a general lack of data for nonstate actors. While there are efforts to rectify this (e.g. Cunningham, Gleditsch and Salehyan 2013; Fjelde and Nilsson 2012), the actors contained in these data sets are restricted compared to the population of nodes on the present network. I therefore limit these models to include only attributes associated with states.

For each submodel, I also include a set of network dependence terms that do not operate across different types of ties. I focus primarily on network structures that capture the tendency for conflicts to cluster, as this remains a persistent question in conflict studies (Braithwaite 2005; Gleditsch 2007; Buhaug and Gleditsch 2008; Gleditsch, Salehyan and Schultz 2008; Gibler and Braithwaite 2013). Clustering in conflict studies commonly refers to the general tendency for actors in proximity to other actors involved in conflict to also become involved in conflict (Buhaug and Gleditsch 2008; Gleditsch 2007). The most localized form of a conflict cluster is when one actor is simultaneously engaged in conflict with two other actors. In network terms, this local structure is called the “two-star”. When the count of two-stars on a network is specified as an ERGM term, it can be used to capture the tendency for ties to cluster around a small number of nodes (Koskinen and Daraganova 2013). Cranmer and Desmarais (2011) find this to be a generative feature of the MID conflict network. In many social networks however, the “popularity” of an actor is likely to eventually plateau as opposed to increase unconstrained. Alternatively put, while there will be clustering in the form of multiple conflicts sharing the same center node, each additional tie formed by this node becomes less likely than the previous one. This is an oft-observed tendency for social networks (Koskinen and Daraganova 2013), and is likely to hold for conflict networks as well, given that piling-on is subject to diminishing gains and geographical constraints.

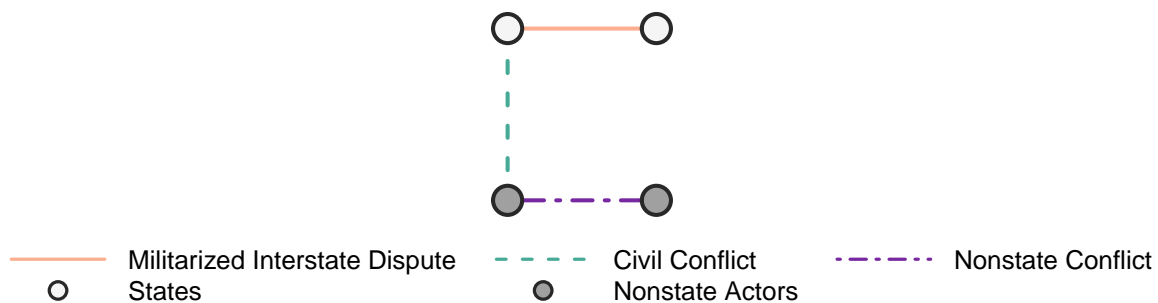
To account for this, I include the alternating  $k$ -star term, which is an extension to the two-star count that attenuates the tendency of tie-clustering as clustering increases (Snijders et al. 2006; Robins et al. 2007). In addition to its theoretical appeal, the alternating  $k$ -star terms improve model fit and reduce their tendency for degeneracy in the estimation procedure (Snijders et al. 2006). Interpretation of the estimated coefficients of these alternating  $k$ -star terms are similar to that of the two-star; a positive coefficient indicates clustering around a

relatively small number of high-degree nodes and a negative coefficient indicates a relatively equal distribution of conflict across the system.

The steps described above yield the layer independence model. Recall from the ChemG multiplex application that if the system is modeled without dependence across different sets of ties (i.e. different sets of ties are not constrained to have the same likelihood of being observed and there are no cross-layer dependence terms), estimated coefficients for such a model will be identical to the three sets of ties modeled in three separate models. As before, the layer independence model will fit the observed system poorly if interdependence exists across the three different conflict subsystems.

From here, I specify the cross-layer dependence model by replacing the exogenous conflict activity terms (e.g. civil war activity on the MID network) with two alternating  $k$ -star terms that are computed over two sets of ties at the same time. These are cross-layer dependence terms that respectively capture the tendency for MIDs and civil conflicts to cluster, and the tendency for civil and nonstate conflicts to cluster, both subject to plateauing in the same manner as within-layer alternating  $k$ -star terms (Wang, Robins and Matous 2016). As noted in the discussion of the Levantine conflict network, to specify one set of conflict as predictors of another requires assumptions about causal direction that are rarely justified in conflict studies. The cross-layer clustering terms allow me to model interdependence as network effects, thereby relaxing these assumptions.

Figure 9: Local Network Structure for Three-way Clustering



Finally, to account for the potential for all three types of conflicts to cluster together, a generative feature that cannot be captured without the use of cross-layer dependence terms, I introduce a three-way clustering term computed over all three tie sets. This particular clustering term captures the tendency for three-way clustering by counting the number of edges incident to at least one cross-layer three path, which is shown in Figure 9. The term I introduce here differs from the conventional approach of modeling higher-ordered clustering, which is by counting cross-layer three path structures (Wang et al. 2013). Similar to

the alternating  $k$ -star terms described above, this term downweights additional clustering ties beyond the first, and avoids the tendency for model degeneracy associated with more conventional structure counts (Snijders et al. 2006).

## C Global Conflict Network Model Fit Comparison

In this section, I present results demonstrating that the cross-layer dependence model better fits the observed network compared to the layer independence model. Given the substantive focus of this application, I evaluate how closely simulated networks from each model resemble how interstate conflict dyads are distributed based on how they cluster with civil and nonstate conflicts. I refer to these clustering scenarios as the sub-state level strategic environment facing states that are involved in interstate conflict. These strategic environments are stylistically illustrated in Figure 1, and Table 1 provides one example of each type during the post-Cold War period.

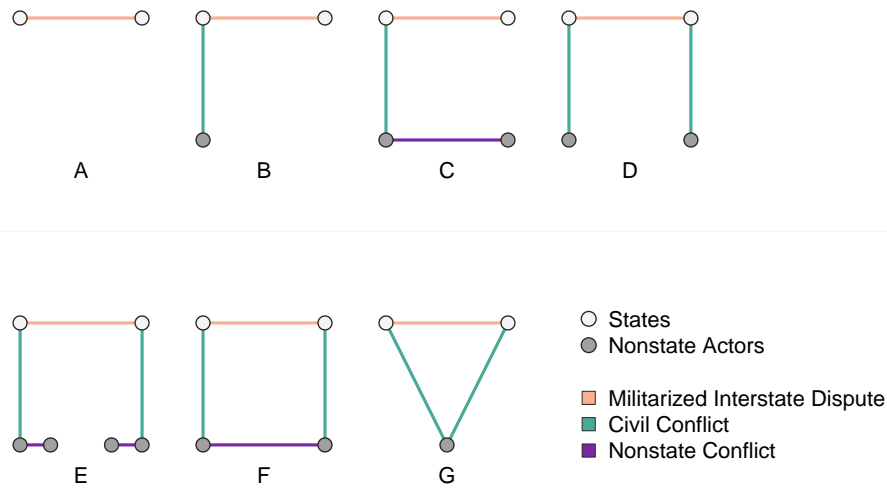
For each simulated network, I take the set of MID ties on the multilayer network, partition them into subsets according to the strategic environment under which they occur, and count the size of these subsets. For example, for an interstate conflict dyad to be in environment  $D$ , both states have to be involved in at least one civil conflict with none of their civil conflict partners further fighting nonstate wars. When partitioning interstate conflict ties into these strategic environments, I treat the presence of multiple ties the same as one tie, meaning that regardless of how many civil conflict partners two states have, as long as none of them are further engaged in nonstate conflict, the interstate dyad faces environment  $D$ . Finally, all conflict dyads that share civil conflict partners are classified as  $G$  regardless of whether these nonstate conflict partners are involved in other conflicts.

Table 1: Examples of Strategic Conflict Environments

	MID Dyad		Nonstate Actors	
A	North Korea	South Korea		
B	United States	Iran	Democratic Party of Iranian Kurdistan	
C	Ecuador	Peru	Túpac Amaru Revolutionary Movement	Shining Path
D	Colombia	Nicaragua	FARC	Contras
E	Israel	Lebanon	Hezbollah	Forces of Michel Aoun
F	Iraq	Turkey	Patriotic Union of Kurdistan	Kurdistan Worker's Party
G	Afghanistan	Russia	National Islamic Front of Afghanistan	

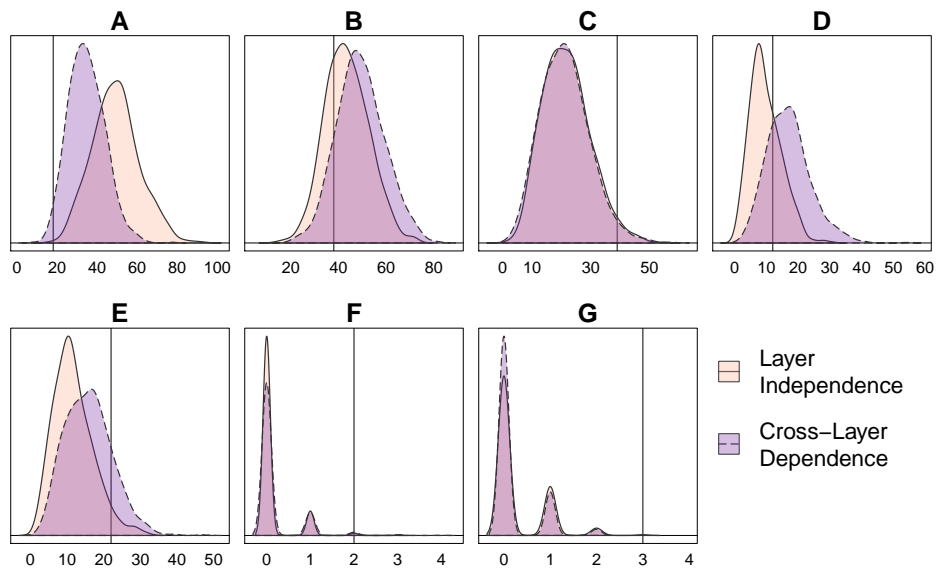
I simulate 1000 networks from each model and examine the resulting distributions, which

Figure 1: Census of Strategic Conflict Environments



are visualized in Figure 2. In each subfigure, the simulated distribution of counts of the given strategic environment are colored purple (dashed line) for the cross-layer dependence model and peach (solid line) for the layer independence model. The vertical black line is the value from the observed network. We should expect cross-layer dependence models to yield improvement in fit only if dependence exist and only if these dependences are modeled. Here, it is important to note that the three-way clustering term is the only additional information specified in the cross-layer dependence model that is not available in the layer independence model. The resulting distributions of simulated networks indicate that the inclusion of this additional piece of information either improves or maintains model fit across all strategic environments. Specifically, by including cross-layer dependence terms into the model, I obtain better fit for three of the seven strategic environments, which suggests that the model more closely captures the strategic considerations of states choosing to engage in conflict. In certain environments, both models fit poorly. This can likely be attributed to the sparsity of the submodels for civil and nonstate conflicts. These results read collectively indicate the value of cross-layer dependence models in improving model fit over models constrained to not account for cross-layer dependence.

Figure 2: Comparison of Model Fit for Different Strategic Environments





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